Topologically-Protected Long Edge Coherence Times in Symmetry-Broken Phases

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Introduction

Practical quantum computation requires systems with long coherence times. This has driven recent theoretical interest in the limits and causes of decoherence in quantum many-body systems where, typically, local quantum information is rapidly scrambled. One tactic to store and process quantum information is to use topological edge modes. Combining these with many-body localization, information can be protected for infinite times, even at effectively infinite temperature [1]. Another option is to use prethermalization, some observables retain memory of the initial state for long times before finally reaching equilibrium, leading to exponentially long coherence times [2, 3]. Furthermore, the recent experimental search for quantum spin liquids has led to the proposal of a "proximate spin liquid" in α-RuCl$_3$ [4]. We meld these ideas into a sharply defined "proximate SPT regime", where topological effects leaks across phase boundaries, leading to anomalously long coherence time in nearby phases [5].

Model

Consider a spin-$\frac{1}{2}$ chain $(\sigma_0, \tau_0, \sigma_1, \tau_1, \dots, \tau_{(L/2)-1})$ with $\mathbb{Z}_2^\sigma \times \mathbb{Z}_2^\tau$ symmetry generated by flipping each species. Define

$$\begin{aligned} \hat{H}(x) &= J\hat{H}_{\mathrm{FM},\sigma} + (1-x)\hat{H}_{\mathrm{PM}} + x\hat{H}_{\mathrm{SPT}} + \hat{V} \\ \hat{H}_{\mathrm{FM},\sigma} &= -\sum_i \sigma_i^z \sigma_{i+1}^z \\ \hat{H}_{\mathrm{PM}} &= -\sum_i \sigma_i^x + B\tau_i^x \\ \hat{H}_{\mathrm{SPT}} &= -\sum_i \tau_{i-1}^z \sigma_i^x \tau_i^z + B\sigma_i^z \tau_i^x \sigma_{i+1}^z. \end{aligned} \tag{1}$$

This interpolates between a paramagnet at $x = 0$ and an SPT (i.e. topological) at $x = 1$ — see Fig 2 for the phase diagram. We add generic symmetry-preserving perturbations $\hat{V}$ to break integrability. One expects topological physics to be lost outside the SPT phase, but it extends to the whole shaded region in dynamics. This "proximate SPT regime" is characterized by anomalously long edge coherence times.

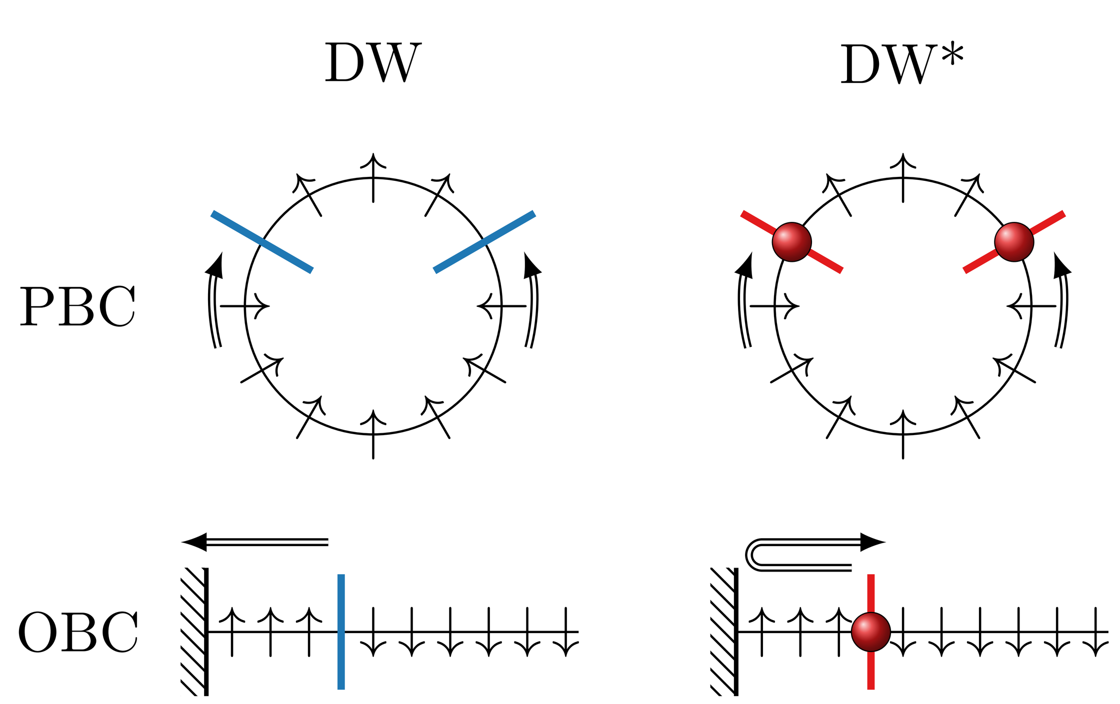

Figure 1: Sketch of the dominant processes that tunnel between the two ferromagnetic ground states. Domain walls (DW) are represented by blue bars, and their decorated counterparts (DW*) are red and carry a $\mathbb{Z}_2$ charge. Under periodic boundary conditions (PBC), the two types of domain walls are equivalent. With open boundary conditions (OBC), however, the decorated domain walls cannot be annihilated at the edges without breaking the symmetry, so will "bounce off" instead. Decorated domain walls are therefore unable to flip the edge spin without breaking the symmetry.

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Quasiparticle Dynamics

The dynamics are governed by the two types of quasiparticles present: regular and "decorated" domain walls [6]. Regular domain walls, from H_{PM}, separate ferromagnetic domains for the σ's, while decorated domain walls, from H_{SPT}, also carry a charge of the $\mathbb{Z}_2^\tau$ symmetry. These are condensed in the PM and SPT phases respectively.

In the bulk, these quasiparticles are equivalent. At the boundary, however, creating a single decorated domain wall is disallowed by the $\mathbb{Z}_2^\tau$ symmetry. This means decorated domain walls cannot flip edge spins without breaking the symmetry, while regular domain walls can. We therefore expect much longer edge coherence times for $x = 1$ relative to $x = 0$ in the ferromagnetic phase. Our key observable is therefore the edge autocorrelation at temperature T, $C_T(t) = \mathrm{Re}\,\langle \sigma_0^z(t)\sigma_0^z(0)\rangle_T$. Its decay is characterized by the coherence time, τ.

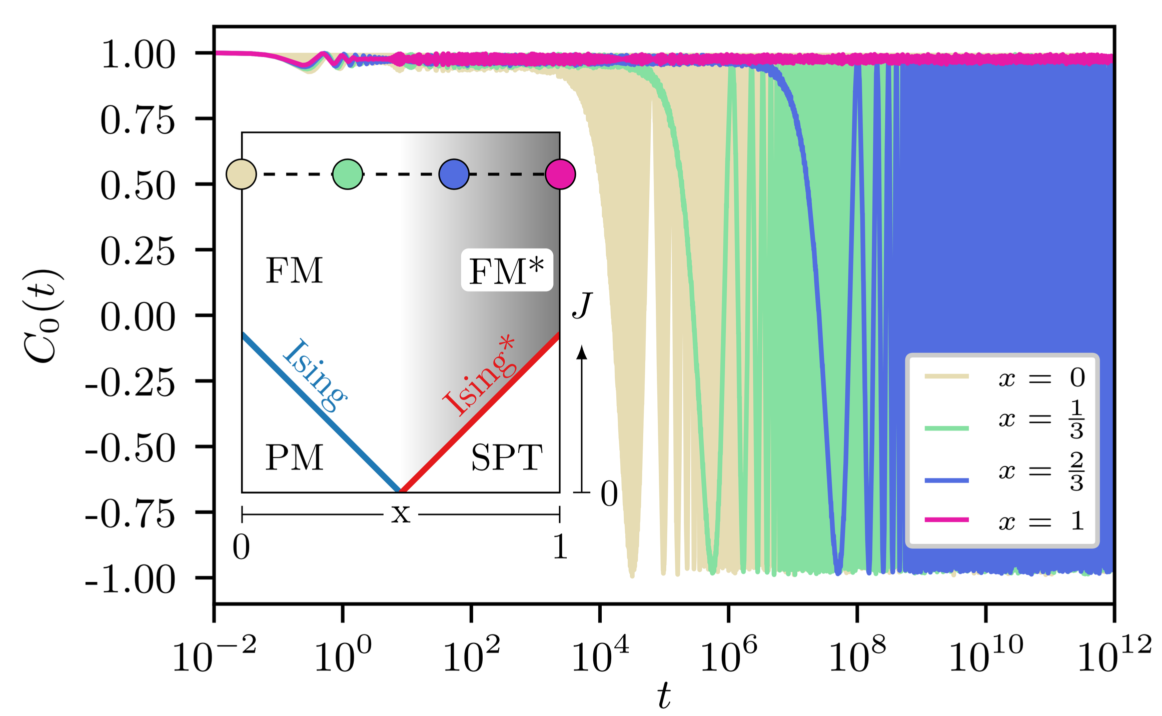

Figure 2: Autocorrelation of the edge spin at zero temperature computed with exact diagonalization (ED) for 14 spins and OBC. Non-zero parameters are $(J, B) = (5.2, 1.274)$. *Inset:* Sketch of the phase diagram for Eq. (1) as a function of x and J. Phases are described in the text. The location of the dots corresponds to the data by color.

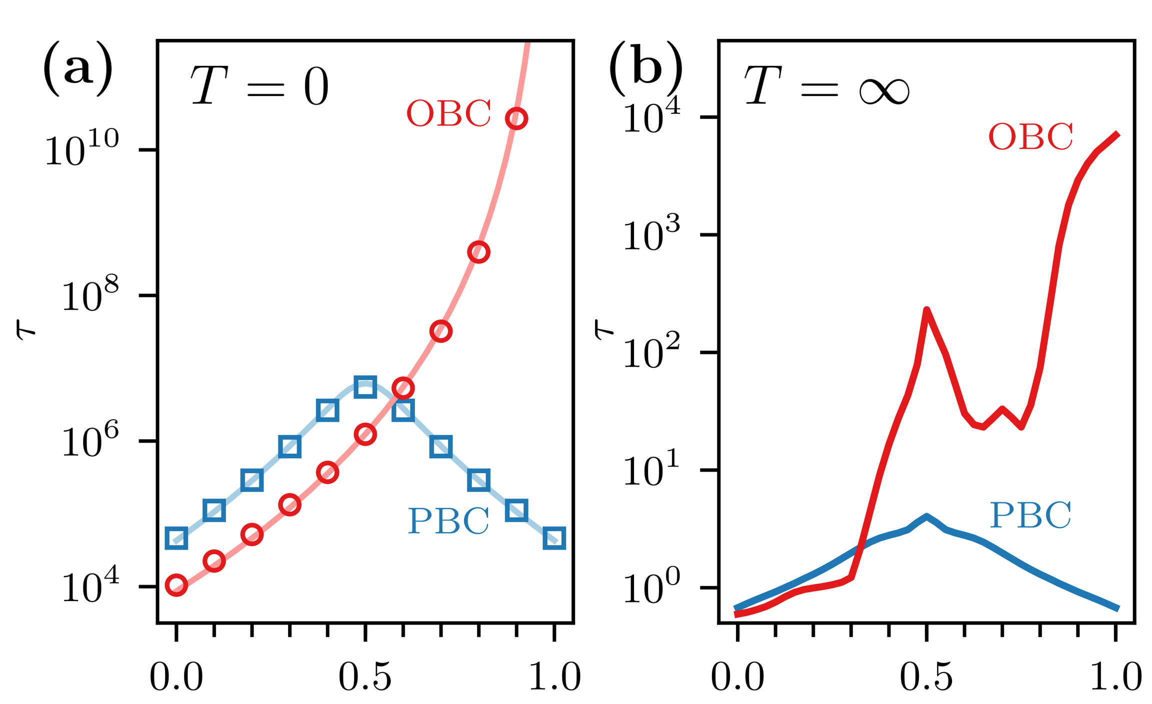

Figure 3: **($\mathbf{T = 0}$)** Comparison of the coherence time (data) with its analytical prediction (lines) Data is computed on 14 spins via ED with parameters $(J, B) = (5.2, 1.274)$. **($\mathbf{T = \infty}$)** Comparison of coherence times for OBC and PBC at infinite temperature on 14 spins. The general trends are the same as at $T = 0$. It was checked that the model is not integrable due to $\hat{V}$ [5].

References

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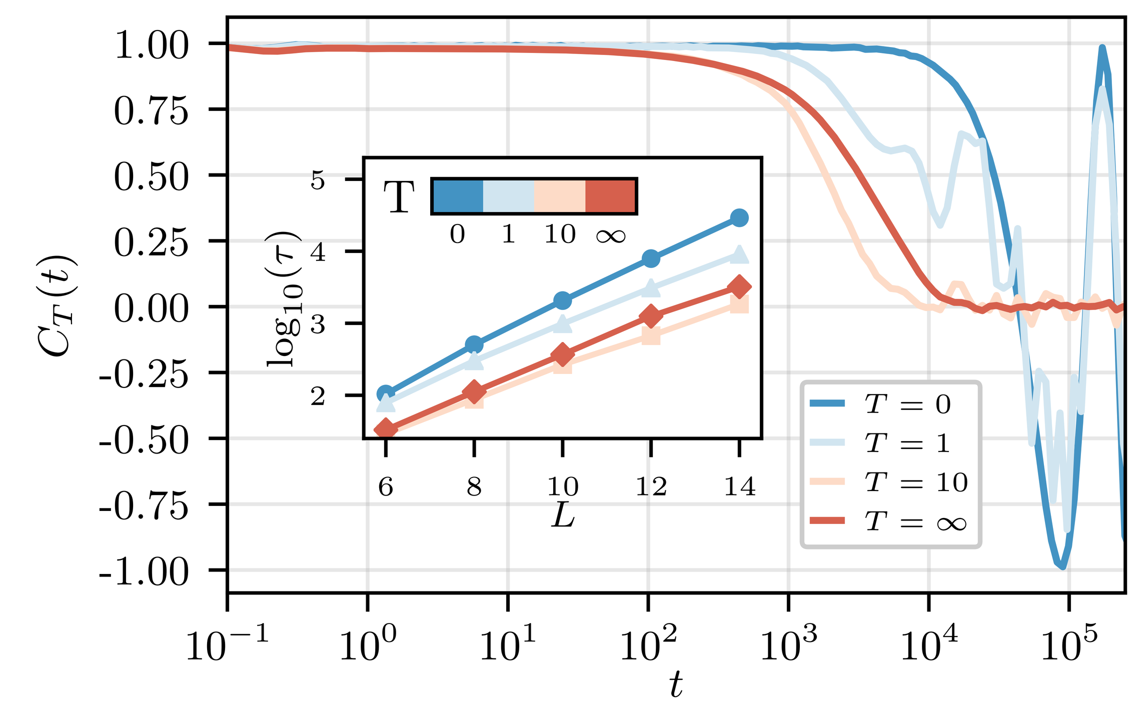

Figure 4: The autocorrelation $C_T(t)$ at four different temperatures as a function of time at $x = 1$. *Inset*: Coherence time as a function of the number of spins at four temperatures. The Hamiltonian is perturbed by $\hat{V}$ to avoid integrability and fine-tuning [5].

$T = 0$ Coherence

At zero temperature, the coherence time τ is controlled by the Rabi oscillation between the two ferromagnetic ground states: $\tau \sim 1/\Delta E$. We calculate this in perturbation theory.

With PBC, the lowest order tunneling process is order $L/2$: two domain walls are created, propagate around the system, and annihilate each other (Fig. 1). This can occur for either regular or decorated domain walls, so

$$\Delta E_{\mathrm{PBC}}(x) \propto \Delta E_{\mathrm{DW}} + \Delta E_{\mathrm{DW}^*}, \tag{2}$$

where $\Delta E_{\mathrm{DW}} = \left(\frac{1-x}{4(J+xB)}\right)^{L/2}$ is the contribution for regular domain walls and $\Delta E_{\mathrm{DW}^*}(x) = \left(\frac{x}{4(J+(1-x)B)}\right)^{L/2}$ is the contribution for decorated domain walls. (Symmetric under $x \leftrightarrow 1-x$.)

For OBC, decorated domain walls cannot flip an edge spin, so

$$\Delta E_{\mathrm{OBC}}(x) \propto \Delta\tilde{E}_{\mathrm{DW}}, \tag{3}$$

where the tilde signifies that the regular domain wall contribution is slightly modified compared to PBC: $\Delta\tilde{E}_{\mathrm{DW}} = \frac{1}{1-x}\left(\frac{1-x}{2(J+xB)}\right)^{L/2}$. Fig. 3 (a) shows that Eqs. (2) & (3) accurately predict the coherence times. We have checked that adding perturbations does not change this pattern, though it does reduce the divergence to a finite value. We therefore see the predicted enhancement of coherence in the proximate SPT regime at $T = 0$.

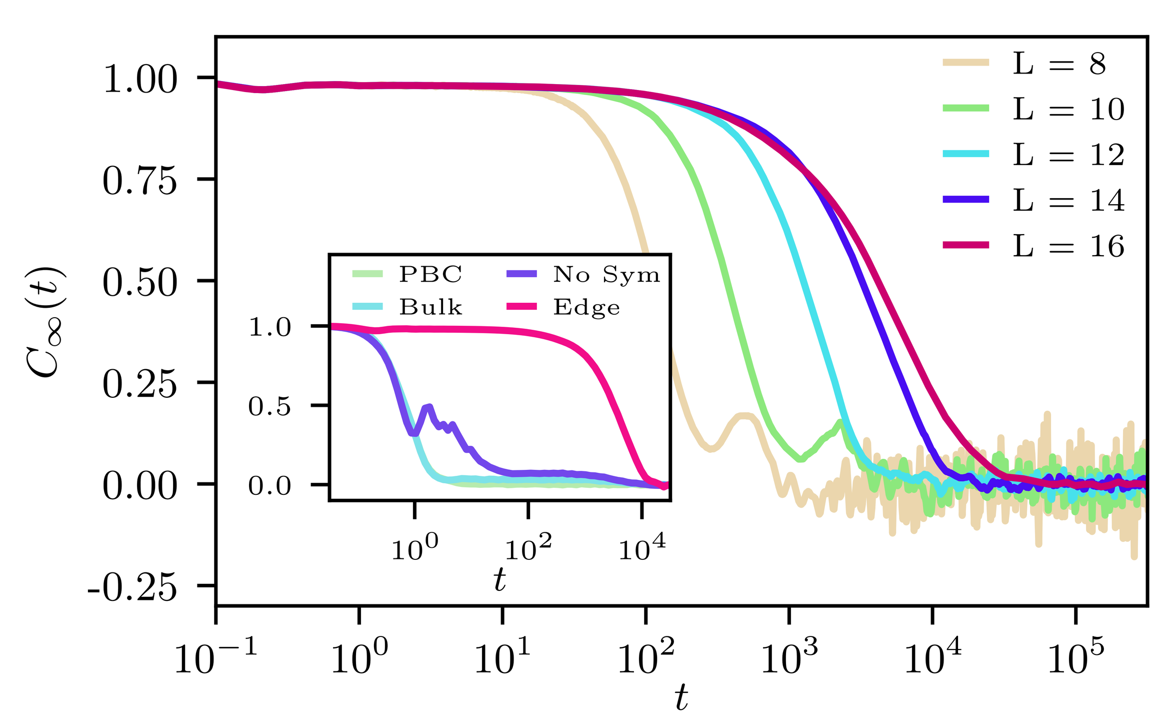

Figure 5: Autocorrelation $C_\infty(t)$ at $x = 1$ and $T = \infty$ under OBC and varying system size. $C_\infty(t)$ remains close to one for a time τ until it drops to its thermal value of zero, and τ increases exponentially with system size until reaching saturation around $L = 16$. *Inset:* The same autocorrelation $C_\infty(t)$ under various conditions on 14 sites. 'Edge' is same as in the main panel, 'bulk' corresponds to $\sigma_{L/4}^z$, 'PBC' corresponds to periodic boundary conditions, and 'No Sym' corresponds to a system where the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry was broken explicitly with edge perturbations $\sigma_0^x\tau_1^z$ and $\sigma_0^y\tau_1^z$.

$0 < T \le \infty$ Coherence

Surprisingly, the proximate SPT regime persists to all temperatures $0 \le T \le \infty$ (see Figs 4, 5, 6). Naively, one expects that finite density of both types of domain walls would cause the coherence to vanish. However, we may rewrite the Hamiltonian as

$$\hat{H} = -B\left[x\hat{N}^* + (1-x)\hat{N}\right] + \hat{V}_p, \tag{4}$$

where $\hat{N}^* = \sum_i \sigma_i^z \tau_i^x \sigma_{i+1}^z$, $\hat{N} = \sum_i \tau_i^x$ and $\hat{V}_p$ contains $\mathcal{O}(1)$ terms that are independent of B. Any process which flips the edge spin must change the expectation of the $\hat{N}^*$ operator, and such processes are exponentially suppressed due to the so-called ADHH theorem [7]. We therefore expect a coherence time $\tau \sim e^{Bx}$, as seen in Fig. 3 (b).

The proximate SPT regime is not fine-tuned in any obvious way. Fig. 6 shows its level statistics are far from integrability, and its density of states has no quasi-periodic or oscillatory structure. As expected [2, 3], the coherence time increases with system size before saturation at $L = 16$. We have checked that the Hamiltonian is free from accidental resonances.

This enhancement of the coherence is "topological", since only the coherence of the edge is exponentially enhanced and, unlike previous applications of the ADHH theorem [2, 3], it is also symmetry-protected (inset to Fig. 3). This provides a clear example of (prethermal) SPT physics even at infinite temperature, in a regime where the protecting symmetry is spontaneously broken at zero temperature.

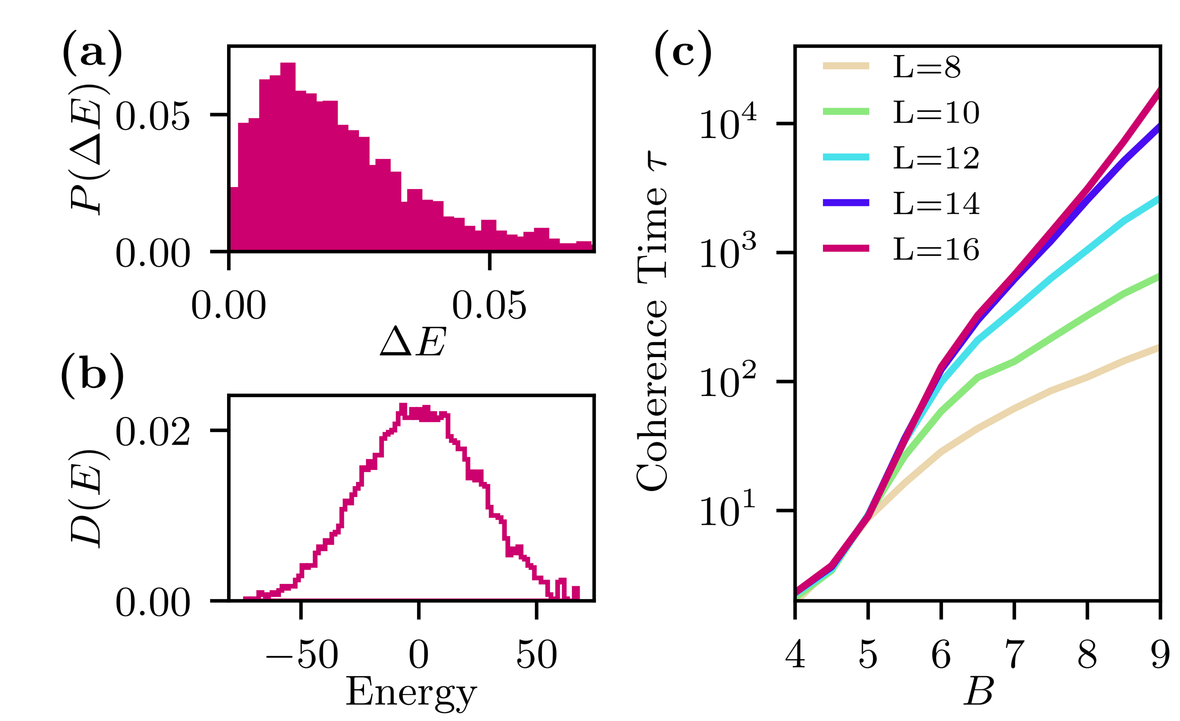

Figure 6: **(a)** Histogram of the differences in adjacent energy levels and **(b)** normalized density of states in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ even/even sector on 16 spins. **(c)** Dependence of the coherence time for $x = 1$ on B, which sets the energy scale for the τ spins. One can already see the saturation with system size for smaller values of B. Numerical details are given in the Suppemental Material.

Conclusions

We have demonstrated the existence of a proximate SPT regime, characterized by anomalously long edge coherence times. The key to the model's dynamics is the behavior of its two species of quasiparticles: regular and decorated domain walls. The decorated domain walls, which are inherited from the SPT phase, cannot be created or annihilated near the edges of the system without breaking the symmetry, giving rise to a dramatic increase in edge coherence. In the special case of zero temperature, we confirmed the quasiparticle picture within perturbation theory. We have shown that the phenomena is robust; the enhancement of edge coherence is stable to symmetry-preserving perturbations, integrability-breaking perturbations and, via prethermalization, survives at all temperatures.